Functional Property

$$
\begin{aligned}
&\mathit{isFunctional}(r) \Leftrightarrow \\
&\quad \forall s, t1, t2 \bullet \\
&\qquad (s \in S \land t1 \in T \land t2 \in T) \\
&\qquad \Rightarrow \\
&\qquad ((s, t1) \in r \land (s, t2) \in r \Rightarrow t1 = t2)
\end{aligned}
$$

Q: Smallest relation satisfying the functional property.

Q: How to **prove** or **disprove** that a relation r is a function.

Q: Rewrite the functional property using **contrapositive**.

Partial Functions vs. Total Functions

$r \in S \nrightarrow T \Leftrightarrow (\ \text{isFunctional}(r) \wedge \text{dom}(r) \subseteq S\ )$

$r \in S \rightarrow T \Leftrightarrow (\ \text{isFunctional}(r) \wedge \text{dom}(r) = S\ )$

Exercise: Visualize $S \nrightarrow T$ vs. $S \longrightarrow T$

e.g., $\{\ \{(2, a), (1, b)\}, \{(2, a), (3, a), (1, b)\}\ \} \subseteq \{1, 2, 3\} \nrightarrow \{a, b\}$

e.g., $\{(2, a), (3, a), (1, b)\} \in \{1, 2, 3\} \rightarrow \{a, b\}$

e.g., $\{(2, a), (1, b)\} \notin \{1, 2, 3\} \rightarrow \{a, b\}$

e.g., $\{(2, a), (1, b), (3, a), (1, a)\} \notin \{1, 2, 3\} \rightarrow \{a, b\}$

Relational **Image** vs. Functional **Application**

A function is a relation.

$f \in \{1, 2, 3\}$ ⇸ $\{a, b\}$

$f = \{ (3, a), (1, b) \}$

Exercises:

$f[\{3\}] =$

$f[\{1\}] =$

$f[\{2\}] =$

Modelling Decision: Relations vs. Functions

An organization has a system for keeping **track** of its employees as to where they are on the premises (e.g., ``Zone A, Floor 23''). To achieve this, each employee is issued with an active badge which, when scanned, synchronizes their current positions to a central database.

Assume the following two sets:

- *Employee* denotes the **set** of all employees working for the organization.
- *Location* denotes the **set** of all valid locations in the organization.

Is $where\_is \in Employee \leftrightarrow Location$ appropriate?

Is $where\_is \in Employee \rightarrow Location$ appropriate?

Is $where\_is \in Employee \nrightarrow Location$ appropriate?